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SEMI-ANNUAL STATUS REPORT ON RESEARCH PROGRESS

NASA Research Grant NsG-2-59
April 1, 1966, to September 30, 1966

During the subject reporting period, faculty members have derived support from NASA Research Grant NsG-2-59 as follows:

1. Professor Frederick J. Beutler was supported on a 25% basis during the months of April and September, and also a part of May. Over the summer, he received support for 20 days.

2. Professor Lawrence L. Rauch was supported on a 15% basis from April 1 through May 21, on a 25% basis from May 21 through July 21, and on a 25% basis from August 25 through September 30.

3. Professor William L. Root did not receive grant support from April 1 to August 31, inclusive; thereafter, he has been supported on a half-time basis.

In addition, approximately half-time support was furnished Mr. Robert Bayma and Dr. Stuart Schwartz, who were working on their doctoral dissertation research under the direction of Professors Rauch and Root, respectively.

The research by Professor Beutler on stationary point processes has continued. Together with Dr. O. Leneman of Lincoln Laboratories, he has studied the theory and applications of such processes. Some of the theory, and in particular those aspects most suitable to applications, has been published as "Random Sampling of Random Processes: Stationary Point Processes," Information and Control, 9 (1966), pp. 325-346 by Beutler and Leneman. "Random Sampling of Random Processes: Impulse Processes," Information and Control, 9 (1966), pp. 347-363, by Dr. Leneman, has also appeared. The latter work represents research in part accomplished as a doctoral student with NASA Research Grant support. A partial summary (from the abstract of the paper) follows:

"The concept of random impulse trains is useful as it facilitates statistical studies on discrete pulse modulated control and communication systems. As a result, this paper discusses the improper random

process

$$s(t) = \sum_{-\infty}^{\infty} \alpha_n \delta(t - t_n)$$

where the impulse occurrence times t_n constitute a stationary point process and where the intensity modulating coefficients α_n constitute a stationary random process, independent of the point process $\{t_n\}$. ...the autocorrelation function and the spectral density function of the impulse process are evaluated."

Yet another publication representing research supported by the grant is Professor Beutler's "Error-free Recovery of Signals from Irregularly Spaced Samples," SIAM Review, 8 (1966), pp. 328-335. Reference is made to the preceding status report (02905-15-P), in which the results presented by this paper are described in some detail.

The work of Professor Rauch with James K. Strozier on optimum PCM demodulators has been completed in accordance with the program described on pages 9 and 10 of the proposal ORA 66-728-PB1 submitted in December 1965. A complete report[*] "The Use of Statistical Dependence Between Data Samples in Binary PCM Demodulation" is being prepared and should be available about the end of 1966. The results show that in typical cases, by the use of practical techniques, the received signal strength requirements for PCM can be reduced two to six decibels below those of the bit-by-bit demodulators presently used. An abstract of the work follows:

"When a time-continuous waveform, not sharply bandlimited, is sampled at a rate sufficiently high to provide a suitably small interpolation error for the reconstruction process, there will be large correlation between adjacent and nearby samples. When these samples are coded in binary PCM for transmission, there results a high statistical dependence between PCM words. This thesis is concerned withPCM demodulators which take advantage of this known "a priori" dependence to produce an optimum or improved estimate of the signal. The channel noise is considered to be additive only, and to be white, bandlimited, Gaussian noise. The data samples are assumed to come from a stationary Gaussian random process with specified correlation between samples, and independent from the channel noise.

"A Monte Carlo simulation on a high speed digital computer is used to evaluate the performance of three optimal demodulation schemes, the minimum error-probability demodulator, the minimum mean-absolute-error demodulator, and the minimum mean-square-error demodulator. The evaluation is done for 3-bit and 6-bit words and for one, two, and three (3-bit words only) words considered in each demodulation. The results establish that a significant improvement over currently used demodulators is possible in the mean-absolute error or root-mean-square error for the optimal

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*Doctoral Thesis of James K. Strozier

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demodulators, particularly for high correlation coefficients and for several words considered in each demodulation. The results indicate that 3-bit data can be used in a computer simulation to accurately predict trends when larger size words are used.

"The optimal demodulators are impractical to build due to their complexity. Three suboptimal schemes of using high statistical dependence between data samples are evaluated and compared with the optimal demodulators. The first scheme was proposed by Earl F. Smith (Minimum-Error Demodulation of Binary PCM Signals, Ph.D. Thesis, University of Michigan, Ann Arbor, Michigan, 1963) and weighs the possible transmitted PCM words by the conditional probability of the ith PCM word given the received signal in the ith word time and the previously demodulated words. It is simpler than the optimal demodulators and gives only fair performance. The second scheme uses bit-by-bit demodulation, where the decision is made on the basis of a weighted average of the particular bit along several words. The performance of this demodulator showed significant improvement over presently used demodulation schemes and for five words was better than the two-word optimal demodulator for very high correlation coefficients. The one-word optimal demodulator weighs the possible transmitted PCM words by the conditional probability of the ith PCM word given the received waveform in the ith word time. The third suboptimal scheme evaluated replaces the received waveform at the input of the one-word optimal demodulator by the set of weighted bits from the second suboptimal scheme. This demodulator gives the best performance of the three suboptimal schemes evaluated. The performance of the first and third suboptimal demodulator was evaluated by a Monte Carlo, high speed, digital computer simulation, while the performance of the second one was evaluated directly.

"The results are generalized to arbitrary bit waveforms, and the generalization to other than Gaussian data, white noise, and additive noise is discussed briefly.

"The major conclusions are that high correlation between PCM data samples can be taken advantage of to significantly improve the demodulation process, and that this improvement is possible in practical demodulators. Also, the technique of a Monte Carlo simulation on a high speed digital computer offers a valuable means of evaluating the performance of complex demodulators when direct evaluation is impossible or impractical."

The work on improved analog demodulation with Professor Rauch's doctoral student, Robert W. Bayma, is still in the exploratory stage and will be pursued during the next reporting period.

With the support of the research grant, Professor Root has continued work on a report which he started while he was with the Radar Laboratory, Institute of Science and Technology, The University of Michigan, during the summer. This report contains the beginnings of a theory of achievable

communication rates in environments where the signal is distorted by influences which cannot reasonably be regarded as probabilistic in nature. An example is a situation where radio waves are transmitted through a medium whose properties are not very well known and which may be changing with time. The simple fundamental question which is asked is: how many "distinguishable" messages can be transmitted in a given time from a specified signal space? The word "distinguishable" is taken to mean that the received signals are farther apart than some fixed tolerance, according to some measure of distance. Some estimates of number of signals and rate of possible transmission have been obtained. A briefer, more mathematically oriented version of the same subject is being written by Professor Root jointly with Professor Reese Prosser of Dartmouth College.

During the period Professor Root drew no support from the grant he was actively involved in the supervision of the doctoral research of Dr. Stuart Schwartz, who was supported by the grant. Dr. Schwartz completed his work during this time, and early in September passed the examination and had his dissertation accepted. His work, entitled, "An Empirical Bayes Technique in Communication Theory," is summarized below:

"This thesis is concerned with an empirical Bayes procedure and its application to communication theory. The communication problem is one in which a sequence of information bearing signals is either assumed to be a stationary random process or distorted by a stationary random process. In either case, the underlying probability structure is unknown. The message sequence is then added to correlated gaussian noise. The statistical inference problem is to extract information from each member of the observation sequence, i.e., make a decision as to the presence of a particular signal. The empirical Bayes procedure utilizes all past observations to obtain consistent estimates of the unknown distributions or related quantities. These estimates are then used to form a sequence of test functions which is evaluated using only the present observation. It is shown that the sequence of test functions converges to the test function one would use if all distributions were known and if the observations were independent. For a minimum probability of error criterion, the resulting difference in error probabilities is dominated by a quantity proportional to the mean-square error in the estimate of the test function.

In particular, we consider the class of problems where the marginal density function of an observation is the convolution of a gaussian density function and an unknown distribution, $f(x) = \int g(x-z;\sigma)\,d\alpha(z)$. By suitably interpreting $\alpha(z)$, a variety of communication problems are included. Much of this study is concerned with obtaining consistent estimates of $f(x)$ given the sequence of dependent, identically distributed random variables $X_i = N_i + Z_i$, $i = 1, \ldots n$. Three techniques are presented: a kernel method which is similar to the procedure used for estimating a spectral density, an orthogonal expansion for $f(x)$ in Hermite functions, and an eigenfunction representation obtained by solving an eigenfunction problem

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associated with the integral equation for $f(x)$. For all three methods, we calculate the bounds on the mean-square error in the estimate of $f(x)$. A typical result is: if the autocorrelation function of the gaussian noise is absolutely integrable and eventually monotonically decreasing, and if the sequence Z_i is M-dependent, the rate of convergence of the estimates is the same as in the case of independent observations. The rate is $O(1/n^{4/5})$ for the kernel method. For the orthogonal expansion, with the r-th absolute moment of Z finite, the rate is $O(1/n^{(r-2)/r})$. With the eigenfunction representation, we estimate a quantity related to $f(x)$ and obtain the rate $O(\ln^2 n/n)$. The techniques are then extended to the case of estimating a k-variate density function $f(x_1 \ldots x_k)$.

These results allow us to bound the rate of convergence of the risk incurred using the empirical procedure in a number of communication problems. The problems considered are: communication through an unknown, stationary, random channel when learning samples (channel sounding signals) are available, communication through an unknown random multiplicative channel, and the transmission of known signals with unknown a priori probabilities."